On Russell's Derivation of Quantum Mechanics

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1 Point of Departure

In his article *Why Occam's Razor*[1] and in appendix D of his book *Theory of Nothing*[2], Russell presents a derivation of the postulates that underly quantum mechanics based on the theory of the Everything ensemble. In usual treatments of quantum mechanics that can be found in various textbooks, these postulates aren't justified on any deeper level. Though, there have been considerable efforts (mostly linked to the *Everett interpretation*, also called *many-worlds* or *relative state interpretation*) to explain the apparent validity of the postulates describing the collapse of the wavefunction starting from the no-collapse postulates. Recent contributions have been published by Wallace[3] and Zurek[4]. But Russell goes even much further: He also derives the core of quantum mechanics, its no-collapse postulates, using the theory of the Everything ensemble and a few assumptions.

If Russell is right, then his derivation is a great and to date unrivalled highlight of our efforts for justifying the theory of the Everything ensemble. Aspects of the structure

[1] Russell K. Standish, *Why Occam's Razor*, arXiv.org e-print physics/0001020 (2000).

[2] Russell K. Standish, *Theory of Nothing*, BookSurge Publishing, or online at http://www.hpcoders.com.au (2006).

[3] David Wallace, *Quantum Probability from Subjective Likelihood: improving on Deutsch's proof of the probability rule*, arXiv.org e-print quant-ph/0312157 (2003).

[4] Wojciech Hubert Zurek, *Relative States and the Environment: Einselection, Envariance, Quantum Darwinism, and the Existential Interpretation*, arXiv.org e-print 0707.2832v1 (2007).

of our world are explained by reason alone without referring to experiments—this could be the first great achievement of what I call *rationalist physics*. His work induces Russell to be enthusiastic: referring to Feynman's famous statement that "nobody understands quantum mechanics", Russell writes in chapter 7 of his book: "I can now say that I *understand* quantum mechanics." and he summarizes "Quantum mechanics is simply a theory of observation!"

The significance of Russell's claim cannot be overrated. And I do hope that he is right. Nonetheless, I elaborate a thorough criticism of his derivation. If Russell can disprove my objections (and I hope he will), my criticism will contribute to a clarification of several issues. If my criticism holds, then it is up to all of us to improve Russell's approach or to suggest completely new ideas. So, I invite all of you to participate actively in the discussion that will follow.

I will outline Russell's derivation step by step. My presentation sticks closely to appendix D of Russell's book. I slightly changed notations in order to avoid confusions.

2 Russell's Derivation

2.1 The postulates of quantum mechanics

First of all, I address the postulates of quantum mechanics that Russell derived:

1. A physical system is associated with a Hilbert space V, and vectors $\psi \in V$ are associated with states of the system.

2. The state vector ψ evolves in time according to the Schrödinger equation

$$i\hbar \frac{d\psi}{dt} = H\psi \tag{2.1}$$

   where H is a Hermitian operator on V.

3. An observable is a Hermitian operator A acting on vectors in V, and the outcome on any measurement of the observable A is an eigenvalue a of A. Each (non-degenerate) outcome a has a probability given by (Born's rule)

$$\frac{|\langle \psi, \hat{\psi}_a \rangle|^2}{\langle \psi, \psi \rangle} \tag{2.2}$$

   where $\hat{\psi}_a$ is the related normalized eigenvector.

Comments: Note that the third postulate describes the collapse of the wavefunction. As I have written in section 1, the third postulate can possibly be explained by the first and second postulates. So, it is arguable whether it needs to be directly derived from the theory of the Everything ensemble. Note also that in the literature various versions of the postulates can be found, some of them considerably more complex than the three postulates Russell addresses. But mostly, the differences are merely technical details that should not be regarded fundamental. Though, there's one exception: In order to complete the third postulate, we must add that after the measurement (with outcome a), the system is in the state $\hat{\psi}_a$. But this does not affect Russell's derivation because he derived this result, too.

2.2 Russell's assumptions

Now, I will present Russell's TIME and PROJECTION postulates. Of course, there are also other assumptions that enter into his derivation—they are addressed later.

2.2.1 The TIME postulate

Russell's TIME postulate states that "all observers in the Plenitude must find themselves within a temporal dimension" (chapter 4.3 of his book). He reasons:

> "To observe a bit of information requires comparing two different things. These two different things must be brought together to measure the difference. At very least we need a single topological dimension that separates things, and that the mind of the observer can focus its attention from one spot to another."

Russell emphasizes that these requirements allow many models of time. That's crucial because physics has another conception of time than that of the linear parameter we assume in everyday life. Russell's reasoning becomes a bit clearer in his paper *Why Occam's Razor* where he compares the human mind with a Turing machine (solely as a guideline for formulating the TIME postulate, i.e. without thereby accepting computationalism):

> "A Turing machine requires time to separate the sequence of states it occupies as it performs a computation. Universal Turing machines are models of how humans compute things, so it is possible that all conscious observers are capable of universal computation. Yet for our present purposes, it is not necessary to assume observers are capable of universal computation, merely that observers are embedded in time."

Comments: The TIME postulate is very reasonable. A conscious observer does not only have perceptions—he must be able to analyze his perceptions. This task can (at least approximately) be compared with a computation. When talking about observer moments, I would explain time as follows: For reasoning (the analog is a computation), the observer moment must incorporate "memories" i.e. structures that it interprets as information about preceding observer moments. Thereby, the impression emerges that there is a sequence of observer moments—a temporal dimension. Of course, this temporal dimension is subjective. It is the communication between different observers that gives rise to an intersubjective notion of time.

2.2.2 The PROJECTION postulate

Russell's PROJECTION postulate is linked to his claim that physics is an evolutionary process (chapter 6, page 113):

> "Not only is life an evolutionary process, but physics is too. This requirement leads us to conclude that observer[s] will almost certainly find themselves embedded in a Multiverse structure (providing the variation), observing possibilites turning into actuality (anthropic selection) inheritance of generated information (a form of differential or difference equation that preserves information, depending on the precise topology of time)."

In fact, Russell assigns all three principles (categorized by Lewontin) to physics: variation, selection, heritability. For someone not familiar with the theory of the Everything ensemble, this is a strange statement. But for us, it's a brilliant point of view which is nearly self-evident if we assume the theory of the Everything ensemble to be true:

Starting from theories of the Everything ensemble, the Multiverse structure as well as selection (first person indeterminacy even in the presence of third person determinism) are regarded uncontroversial by the majority of participants in the list. And these two principles lead to the PROJECTION postulate. Let us have a look at the postulate's mathematical formulation:

> "the concept of observer moment [...] refer[s] to the knowledge state of an observer at a point in time. The state vector ψ corresponds to this observer moment." (chapter 7.1, page 118)

In appendix D of his book, Russell even defines $\psi(t)$ to be an observer moment:

> "Observer moments $\psi(t)$ are sets of possibilites consistent with what is known at that point in time, providing variation upon which anthropic selection acts."

The PROJECTION postulate is now formulated as follows: An observer chooses an "observable" A, i.e. a partition of ψ into a discrete set $\{\psi_{\{a\}}\}$ (i.e. $\bigcup_a \psi_{\{a\}} = \psi$ and $\psi_{\{a\}} \cap \psi_{\{b\}} = \emptyset$ if $a \neq b$), where every a is a possible outcome. Discreteness means that the outcomes can be counted: a_1, a_2, ...Furthermore, we postulate that each outcome has a probability $p_\psi(\psi_{\{a\}})$. Russell assumes that the Kolmogorov probability axioms can be applied:

(A1) If A and B are events, then so is the *intersection* $A \cap B$, the *union* $A \cup B$ and the *difference* $A \setminus B$.

(A2) The *sample space* S is an event, called the *certain event*, and the *empty set* $\emptyset$ is an event, called the *impossible* event.

(A3) To each event A, $p(A) \in [0, 1]$ denotes the *probability* of that event.

(A4) $p(S) = 1$.

(A5) If $A \cap B = \emptyset$, then $p(A \cup B) = p(A) + p(B)$.

(A6) For a decreasing sequence $A_1 \supset A_2 \cdots$ of events with $\bigcap_n A_n = \emptyset$, we have $\lim_{n\to\infty} p(A_n) = 0$.

Comments: The informal argument, i.e. the application of Lewontin's first two principles to physics, bases upon ideas which are quite uncontroversial for us. But I will explain why I think that the formalization is very controversial.

I have several objections. The first one concerns the definition of ψ as an observer moment. In physics, the state vector describes a (typically small) system $\mathcal{S}$. Measurement then leads to an entanglement of $\mathcal{S}$ with the (larger) environment $\mathcal{E}$ to which the observers belong. In this setting, it seems to be incomprehensible how an observer moment can be identified with the state vector of the system $\mathcal{S}$: First of all, the observer moment contains a lot of things that have nothing or little to do with the system $\mathcal{S}$. Furthermore, in physics, different observers (i.e. different observer moments) will agree on the state of the system $\mathcal{S}$. Thus, the identification of the state vector with a subjective state of knowledge seems to be dubious.

We could try to solve this problem by defining $\mathcal{S}$ to be the "rest of the universe" as seen by the observer. But this is not the typical situation the postulates of quantum mechanics are usually applied to. Additional effort would be necessary to describe the transition between the two concepts.

Another related objection refers to the *objectivity* of the physical state vector. Physics succeeds in meaningfully applying the postulates of quantum mechanics even to periods of time during which the human race did not exist. This, of course, may not be true for the collapse postulate, but at least for the Hilbert space structure as well as the unitary evolution. If one defines the state vector to be an observer moment, then one must explain this phenomenon.

There is still another very serious (though more subtle) objection that I want to raise. It concerns the use of Kolmogorov's probability axioms. The axioms don't come from the theory of the Everything ensemble, they are mathematics. That means there is no logical necessity to interpret a structure obeying the axioms as "probability" in the intuitive sense. I suggest to develop a theory of probability from our theory of the Everything ensemble (that naturally fits our intuitive conception of probability). Having done this, one can check whether Kolmogorov's probability axioms are satisfied by this new theory. Thus, I want to understand Russell's use of Kolmogorov's axioms as follows: There is a theory of probability that can be constructed out of the theory of the Everything ensemble. So far, we don't have this theory; however, we act on the assumption that this theory satisfies Kolmogorov's probability axioms.

2.3 Projection and addition

The observer moment ψ is an ensemble of possibilites. The set of all these ensembles is called V. So far, V has no algebraic structure but Russell derives that (using adequate constructions) V gets the structure of a Hilbert space. The first step towards this aim is to find linearity.

Russell defines the projection operator $\mathcal{P}_{\{a\}} : V \to V$ where a is a possible outcome:

$$\mathcal{P}_{\{a\}}\psi = \psi_{\{a\}} \tag{2.3}$$

The set of all possible outcomes is called S. Remembering Kolmogorov's probability axioms (that the union of two events gives another event), we can generalize this to

$$\mathcal{P}_A\psi = \psi_A \tag{2.4}$$

where $A \subset S$ is a set of possible outcomes. I define $\psi_A = \bigcup_{a\in A}\psi_{\{a\}}$; in particular, we have $\psi = \psi_S$. This generalization can be used to define an addition

$$\mathcal{P}_{\{a\}}\psi + \mathcal{P}_{\{b\}}\psi = \mathcal{P}_{\{a,b\}}\psi \text{ if } a \neq b \tag{2.5}$$

which can be understood as an addition of observer moments[5]

$$\psi_{\{a\}} + \psi_{\{b\}} = \psi_{\{a,b\}} \tag{2.6}$$

providing some kind of linear structure for V. Applying the definition of our addition to more general cases yields

$$\begin{aligned}
\psi_A &= \sum_{a\in A}\psi_{\{a\}} && (2.7)\\
\psi_{A\cup B} &= \psi_A + \psi_B - \psi_{A\cap B} && (2.8)\\
\psi_{A\cap B} &= \mathcal{P}_A\mathcal{P}_B\psi = \mathcal{P}_B\mathcal{P}_A\psi && (2.9)
\end{aligned}$$

for arbitrary sets $A, B \subset S$.

Now, we consider one single observer moment ψ. Since ψ is the ensemble of all possibilities consistent with what is known, any outcome will also be consistent with

[5] Russell goes on writing down the projection operators. I apologize that I have perferred another notation but I hope that the equations look less confusing now. In particular, I wanted to avoid the emergence of the letters "$\mathcal{P}$" and "P" within the same equation.

ψ ("it encodes information about the whole ensemble", appendix D, page 219). So, ψ corresponds to the certain event, and we use it as a "reference state". From now on, we consider the subset $V_{\psi,S} \subset V$ of observer moments consistent with ψ, related to the observable that defines the set of outcomes S:

$$V_{\psi,S} = \{\psi_A;\ A \subset S\} \tag{2.10}$$

The probability of a set of outcomes $A \subset S$ (i.e. that one of the outcomes $a \in A$ is observed) is $p_\psi(\psi_A)$. Since ψ itself corresponds to the certain event, we have

$$p_\psi(\psi) = 1. \tag{2.11}$$

Applying the axiom (A4), we get

$$p_\psi(\psi_A + \psi_B) = p_\psi(\psi_A) + p_\psi(\psi_B) \text{ if } A \cap B = \emptyset. \tag{2.12}$$

Comments: From my one experience with Russell's paper and his book I know that the reader unfamiliar with the derivation must read this part at least twice to have an idea of what is going on. But the reader may calm down because we have just developed a formalism without much content so far. Instead of working with "projection operators" and an addition of observer moments, one could have used the simple union of sets. The observer moment ψ is the set of all possibilites consistent with what is known, and it is divided into subsets $\psi_{\{a\}}$ consistent with the observation of the outcome $a \in S$ but inconsistent with another outcome $b \neq a$. Then, $\psi_{\{a,b\}}$ is the set of possibilities consistent with outcome a or outcome b, i.e. the union of $\psi_{\{a\}}$ and $\psi_{\{b\}}$. The other constructions have a preparatory character. But at first, they are harmless.

2.4 Linearity

Until now, the addition defined for observer moments hasn't brought us close to a vector space structure. This is to be changed. Russell argues that "observer moments have multiple observers observing them" (appendix D, page 219). For example, we can assume that n observers (n being a natural number) observe ψ and then choose to partition ψ into ψ_A and $\psi_{\bar{A}}$. Under these conditions, Russell interprets the formula

$$p_\psi(n\,\psi_A) = n\,p_\psi(\psi_A) \tag{2.13}$$

as the measure of observer moment ψ_A.

Comments: Equation (2.13) looks very confusing. ψ_A is a set, so what is $n\,\psi_A$ supposed to mean? Besides, p_ψ assigns probabilites, so how can it become greater than 1 which obviously happens for large n? I don't have final answers to these questions. I guess that Russell introduces a new function $\tilde{p}_\psi$ defined as follows:

$$\tilde{p}_\psi(n, \psi_A) = n\,p_\psi(\psi_A) \tag{2.14}$$

This new function does not only depend on ψ and ψ_A but also on the number n of observers partitioning ψ into ψ_A and $\psi_{\bar{A}}$. The new function provides the measure of the observer moment ψ_A, and it can be greater than 1. Mathematically, the expression $n\,\psi_A$ can be defined as the pair (n, ψ_A). I think that this explains the notations Russell uses.

But there remains a very controversial issue. Russell begins with saying that "observer moments have multiple observers observing them". This statement sounds as if Russell belonged to Bostrom's "duplication" camp[6] which I doubt because Russell puts himself into the "unification" camp. So, maybe I misunderstand Russell. But taken literally, he writes that multiple observers have the same observer moment (the same conscious experience) and that, due to this fact, the observer moment has a greater measure (which puts this idea close to "duplication"). I assume that these observers are meant to be situated in different worlds accross the Multiverse. Understanding Russell like this does not only raise the problem of "unification" and "duplication": We must come back to the criticism I wrote down in section 2.2.2. The probabilites physicists measure are measured by many observers within the same world. So, it seems to be very problematic to define everything in such a subjective manner. We will have to explain why this procedure leads to intersubjective results.

Russell now explores the case that different groups of observers are partitioning ψ in a different way (remember that a partitioning of ψ corresponded to the measurement of an observable; so, two different observables are measured). Let n observers partition ψ into ψ_A and $\psi_{\bar{A}}$, and m observers partition ψ into ψ_B and $\psi_{\bar{B}}$. Russell writes

$$p_\psi(n\,\psi_A + m\,\psi_B) = n\,p_\psi(\psi_A) + m\,p_\psi(\psi_B) \tag{2.15}$$

and calls this the "measure of the combined observer moment" $n\,\psi_A + m\,\psi_B$. In his paper *Why Occam's Razor*, Russell explains[7]:

> "The expression $p_\psi(n\,\psi_A + m\,\psi_B)$ must be the measure associated with n observers choosing to partition the ensemble into $\{A, \bar{A}\}$ and observing an outcome in A and b observers choosing to partition the ensemble into $\{B, \bar{B}\}$ and seeing outcome B." (page 8)

Comments: Once again, we would need to reformulate the equation—this time equation (2.15)—in order to give it a mathematically exact sense. But surely, this can somehow be done. The crucial problem that I see with this equation is its interpretation:

What is a "combined" observer moment supposed to be? What is the meaning of its "measure" (note that the measure of a usual observer moment has a natural meaning due to the self-sampling assumption).

Clearly, Russell starts from equation (2.12), page 6, and tries to go on constructing a true linear structure in a way as natural as possible. This leads Russell to the equations (2.13) and (2.15). Subsequently, he tries to interpret the equations. This is a very important task for several reasons: Only the right interpretation will make sure that we find useful definitions of previously unknown objects such as $n\,\psi_A$ and $n\,\psi_A + m\,\psi_B$. Furthermore, we haven't discovered the Hilbert space structure yet. So, we need the right interpretation of what we have written down in order to proceed correctly. That is why my criticism is not supposed to be pedantism.

We have defined ψ to be the set of all possibilites consistent with what is known, so ψ is a set of infinite bitstrings (descriptions). ψ_A is a subset of ψ. So, what kind of mathematical object is $n\,\psi_A$ and what is $n\,\psi_A + m\,\psi_B$?

2.5 Complex numbers

We have achieved some kind of linearity (with natural numbers n and m) but we finally want to obtain a Hilbert space structure where the coefficients are complex numbers. In fact, Russell generalizes equation (2.15) to

$$p_\psi(\alpha\,\psi_A + \beta\,\psi_B) = \alpha\,p_\psi(\psi_A) + \beta\,p_\psi(\psi_B) \text{ where } \alpha, \beta \in \mathbb{C}. \tag{2.16}$$

[6] Nick Bostrom, *Quantity of experience: brain-duplication and degrees of consciousness*, Springer Science+Business Media B.V. (2006).

[7] I changed notations.

Of course, we need this equation in order to have a vector space over the complex numbers. Equation (2.16) ensures this for $V_{\psi,S}$. But let us pause for a moment and look at the step Russell has made. Previously, the coefficients were natural numbers n, m. I still saw open questions linked to the interpretation of the emerging expressions but at least, we thought that n and m somehow counted observers.

Russell argues that equation (2.15) defines a measure. And mathematically, one can imagine various measures. According to Russell, the most general case is a complex measure.

> "More general division algebras such as quaternions or octonians cannot support equations of the form (2.16) without ambiguity. Thus V, the set of all observer moments, is a vector space over the complex numbers." (appendix D, page 219)

He provides an answer to the question why we should expect that the most general measure is in accordance with observation: Occam's razor. Any other case than the most general one would contain additional constraints remaining unexplained. Consequently, Occam's razor persuades us to accept equation (2.16) for the measure of the "combined observer moment".

Comments: First of all, there is a serious unclarity related to the statement that equation (2.16) makes a vector space out of V. We only have an equation restricted to the subset $V_{\psi,S} = \{\psi_A;\ A \subset S\} \subset V$ which depends on the reference state ψ and on the observable defining the set of outcomes S.[8] So, in fact, $V_{\psi,S}$ becomes a vector space over the complex numbers. But I don't see how this result naturally extends to the whole set V. As we The first idea to solve this problem might be to look at the observer moment that corresponds to no knowledge at all, called Ψ, i.e. the set of all possibilites. Then, for every observer moment $\psi \in V$, we have $\psi \subset \Psi$. V, the set of all observer moments thus is the power set of Ψ. Equation (2.16) implies that $V_{\Psi,S}$ has a vector space structure. But in general, we won't have $V = V_{\Psi,S}$, so this structure does not directly extend to the set V (since $V_{\Psi,S}$ depends on the choice of the observable). We could postulate the existence of an observable that partitions Ψ in such a way that each partition $\Psi_{\{\tilde{a}\}}$ contains but a single possibility ($\tilde{a} \in \tilde{S}$). This would finally lead to $V = V_{\Psi,\tilde{S}}$ and consequently to a vector space strucutre of V. Nonetheless, we still have various problems. Is it natural to assume the existence of such an observable? Do ambiguities arise (we have defined the measure etc. with reference to ψ and S instead of Ψ and $\tilde{S}$)? To conclude, it seems that there is still work to be done.

Let us now come to the crucial aspect: the employment of a complex measure. To be sincere, I admit that this is the point where Russell lost me. Before this point, I saw several problems, maybe even difficult ones; nonetheless, the derivation also showed promise. But I have very fundamental doubts as far as the argument in favor of the complex measure is concerned. The argument itself (that Occam's razor is supposed to persuade us to take the most general concept) isn't bad at all, and it may be convincing for some readers. Though, the way I see it, Occam's razor cannot be used like this.

I'll explain: Let us assume that the map p_ψ defined by equation (2.15) satisfies the axioms of a measure[9]. Surely, p_ψ also satisfies the axioms of some other mathematical structure that I

[8] As we use two different partitions of ψ in equation (2.16), I assume that the vector space structure will at least extend such that it doesn't depend on the observable S. So, maybe it extends to some set V_ψ which I cannot define yet because I don't know the exact mathematical definition of the expressions in equations (2.15) and (2.16). But if correctly defined, maybe there is a solution to this. The problem remains to extend the structure from this (yet undefined) set V_ψ to V without giving rise to ambiguities.

[9] I don't really know whether it does because I don't know the mathematical definition of the expression $n\,\psi_A + m\,\psi_B$.

call XY. So, p_ψ is a measure as well as a XY. Why should we reason that p_ψ should take the form of a general measure and not the form of a general XY?

You may argue that a measure (as the name suggests) is closer to what we think p_ψ means. But this is very dubious reasoning. The axioms of a general measure are defined by mathematicians; they invented the concept for mathematical reasons (it was useful in various situations). But there is no fundamental reason why we should expect that this and no other concept is decisive for what we observe in reality.

Maybe not many participants of the Everything list will agree with me when I say that measure and probability in our theory of the Everything ensemble should only be the consequence of *counting*. For example: If there are only two observer moments that will identify my current observer moment as their predecessor, then my subjective probability of each future observer moment (RSSA assumed) is one half. This is what probability and measure *really* are. The associated mathematical concepts are generalizations and specifications of concepts in our everyday or scientific life; they are not that fundamental!

Intuitively, I guess that the use of complex numbers in quantum mechanics is something that has a (so far unknown) deep explanation. When introducing complex numbers simply referring to Occam's razor, I fear that we are overlooking something great and exciting.

2.6 The scalar product and Born's rule

The map p_ψ, now again with its original meaning as a probability distribution on $V_{\psi,S}$, is used to define a scalar product. Firstly, we use the definition of the addition to write

$$\psi = \sum_{s\in S} \psi_{\{a\}}. \tag{2.17}$$

Russell writes that the $\psi_{\{a\}}$ form a basis of V. Then, any two observer moments $\varphi, \xi \in V$ can be expanded:

$$\varphi = \sum_{a\in S} \varphi^a \psi_{\{a\}} \tag{2.18}$$

$$\xi = \sum_{a\in S} \xi^a \psi_{\{a\}} \tag{2.19}$$

with complex numbers φ^a, ξ^a. The scalar product is defined by:

$$\langle \varphi, \xi\rangle = \sum_{a\in S} (\varphi^a)^* \xi^a p_\psi(\psi_{\{a\}}) \tag{2.20}$$

This connection between the scalar product and the probability distribution can easily be reversed to get another expression for the probability which turns out to be identical to Born's rule:

$$\begin{aligned} p_\psi(\psi_{\{a\}}) &= \langle \psi_{\{a\}}, \psi_{\{a\}}\rangle &(2.21)\\ &= \langle \psi, \psi_{\{a\}}\rangle &(2.22)\\ &= |\langle \psi, \hat{\psi}_a\rangle|^2 &(2.23)\end{aligned}$$

where $\hat{\psi}_a = \psi_{\{a\}}/\sqrt{p_\psi(\psi_{\{a\}})}$.

Comments: I cannot see how it can be true that the $\psi_{\{a\}}$ form a basis of V. Let us assume that the observable partitions ψ into N (the set S of outcomes has N elements) subsets

$\psi_{\{a_1\}} \ldots \psi_{\{a_N\}}$, then V has N dimensions if the $\psi_{\{a_k\}}$ form a basis. Another observable may have a set T of outcomes having M elements. The same reasoning as above will lead to the conclusion that V has dimension M. This contradicts the fact that the dimension of a vector space is unique.

Another problem lies in the choice of the reference state ψ. For example, we could have chosen ψ to be an observer moment consistent with only a single possibility. Clearly, this would lead to $\dim V = 1$, and the $\psi_{\{a\}}$ could not form a basis.

The only possible solution that I can identify is the use of the observer moment Ψ corresponding to no knowledge and of the "maximal observable" partitioning Ψ into observer moments containing only a single possibility. Once again, we are concerned with the question whether there are ambiguities in our definitions.

However—despite of the problems that I see—it is a remarkable fact that Russell finds the exact structure of Born's probability rule. I am very hopeful that this proves to be the consequence of a deep truth that underlies the derivation. Unfortunately, there is also an indication that should warn us not to be too enthusiastic: Gleason's theorem says that for a Hilbert space of a dimension greater than 2, the only possible probability measure of a vector associated with a linear subspace is given by Born's rule. I cannot finally judge whether Russell's discovery reveals something very deep or whether it should not be regarded revolutionary in the light of results like Gleason's theorem.

2.7 Schrödinger's equation

Having established that V is a vector space over the complex numbers, Russell uses axiom (A6) to show its completeness: V is a Hilbert space. I do not outline this part of his derivation because I think that it is not controversial.

> "Finally for the second postulate, we need Lewontin's heritability requirement. In between observations, the observer moment may evolve provided information is conserved. We require that $\psi(t')$ can be computed deterministically from $\psi(t)$, for $t' > t$. This can only be true if $\psi(t)$ is analytic at t. The most general equation for computing ψ as a function of time is a first order differential equation:
>
> $$\frac{d\psi}{dt} = \mathcal{H}(\psi). \tag{2.24}$$
>
> $\mathcal{H}$ does not depend on time, as in this picture time is purely a first person phenomenon. We can say that equation (2.24) provides a clock by which an observer can measure a time interval between two observations. Since measurement of time is given by the clock, we adopt the convention that the clock is constant process.
>
> Since we suppose that $\psi_{\{a\}}$ is also a solution of Eq. (2.24) (ie that the act of observation does not change the physics of the system), $\mathcal{H}$ must be linear. The certain event must have probability 1 at all times, so
>
> $$\begin{aligned} 0 &= \frac{dp_{\psi(t)}(\psi(t))}{dt} && (2.25)\\ &= \frac{d}{dt}\langle\psi,\psi\rangle && (2.26)\\ &= \langle\psi,\mathcal{H}\psi\rangle + \langle\mathcal{H}\psi,\psi\rangle && (2.27)\\ \mathcal{H}^\dagger &= -\mathcal{H} && (2.28) \end{aligned}$$

i.e. $\mathcal{H}$ is i times a Hermitian operator. We may write $\mathcal{H} = \frac{H}{i\hbar}$, and substituting this into eq (2.24) gives us

$$i\hbar \frac{d\psi}{dt} = H\psi, \tag{2.29}$$

which is postulate 2, Schrödinger's equation." (appendix D, page 221)

Comments: It is not clear why the conservation of information implies that $\psi(t')$ can be computed deterministically from $\psi(t)$. Furthermore, we are assuming that time is described as a continuous parameter, so time is modelled by the real numbers $\mathbb{R}$. We can imagine a variety of different models but maybe it will in fact turn out that these models will at least approximately lead to the same results.

3 Conclusions

Undoubtedly, Russell's derivation of quantum mechanics is a great achievement. His two assumptions, the Time and the Projection postulates, are very fundamental and natural ideas. The informal arguments Russell provides are indeed good reasons to believe that the basic structure of quantum mechanics can be explained by a theory of the Everything ensemble. Before I saw Russell's derivation for the first time, I thought that the Multiverse from the Everett interpretation of quantum mechanics wasn't but a single world in the ensemble of all possible worlds. But thanks to Russell's work, I am convinced that there is a deep connection between the quantum mechanical Multiverse and the Plenitude.

Skillfully, Russell avoided proclaiming a scientific revolution referring to his derivation: The derivation in its present form has serious problems. I am sure that I misunderstood Russell several times. Sometimes, I tried to explore different possible ways of understanding what he wanted to say. Unfortunately, I have fundamental doubts whether the derivation is successful. But independently of my personal opinion, it is unquestionable that the derivation is not in a form that will finally convince physicists. So, in any case, much work is left.

To sum up, I saw problems concerning

- the subjective definition of ψ as an observer moment which leads to a gap in the explanation why physics arrives at intersubjective results;
- the obviously different uses of ψ in physics (where ψ refers to a typically small system $\mathcal{S}$) and in Russell's derivation;
- the employment of Kolmogorov's probability axioms and general measure theory—I demand that probability and measure are explained by the theory of the Everything ensemble instead of being introduced as purely mathematical concepts;
- the question whether the measure Russell uses favorises Bostrom's duplication thesis;
- the definitions of various mathematical entities;
- apparent ambiguities in the structure of the space V—at first, the structures are only found for subsets.

This long lists of open issues makes it very difficult to establish an open and lively discussion. I'd really enjoy a personal conversation with Russell next to a blackboard—this could help to easily resolve most of the misunderstandings such that we could concentrate on the truely interesting points. Sadly, this is not an option.

I'm looking forward to having an inspiring discussion that will clarify many concepts and that will let us feel the presence of deep truths underlying our theories.